V = volumetric gas velocity, std. cu. ft./min.
v_{50} = relative intensity of pressure fluctuations, %
$\bar{v}_{50}$ = mean relative intensity of pressure fluctuation, %
v_{90} = relative intensity of density fluctuations, (—)
$\bar{v}_{90}$ = mean relative intensity of density fluctuation, (—)
x = distance from bubble, mm.

Greek Letters

μ = mean of distribution
ρ = density, g./cc.
σ^2 = variance of distribution
$\phi(\Gamma)$ = gamma function
$\phi(\chi^2)$ = chi square function

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Prediction of Drag Reduction with a Viscoelastic Model

G. K. PATTERSON and J. L. ZAKIN

University of Missouri, Rolla, Missouri

VISCOELASTIC THEORIES OF DRAG REDUCTION

A number of polymer solutions which show drag reduction in turbulent flow also exhibit elastic properties in laminar flow. This fact led Dodge and Metzner (*1, 2*) to postulate that the anomalous results they obtained in turbulent-friction-factor measurements in carboxymethyl-cellulose (CMC)–water solutions were caused by elasticity. Savins (*3*) concluded that the elasticity of the polymer solutions studied caused the turbulent fluctuations to be damped and that this damping caused drag reduction. Savins did not propose quantitative expressions to describe the damping postulated.

Some evidence of turbulence damping has been experimentally observed by Shaver and Merrill (*4, 5*) and by Meter (*6*) in dye-injection experiments. Meter recognized the need to correlate drag reduction with elasticity in a quantitative manner. Using a cone-and-plate rheogoniometer and a vibrating rod (0.1 to 400 cycles/sec. Birnboim-Ferry apparatus), Meter measured elastic phenomena in Natrosol (hydroxyethyl cellulose), carboxymethyl-cellulose (CMC), and Carbopol (carboxypolymethylene) solutions. He obtained higher elasticities for Carbopol (non-drag-reducing in turbulent flow) than for CMC at infinitesimal shear rates. This anomaly was attributed to gel formation in the Carbopol solutions.

Meter devised an empirical correlation based on the ratio of wall shear stress to the shear stress $\tau_{1/2}$ at $\mu_o/2$ for his Natrosol-solution friction factor data, where μ_o is the zero shear rate viscosity of the solution. This correlation was recognized by Meter to lack generality because $\tau_{1/2}$ does not exist for dilute solutions, some of which are drag reducing. This correlation may also be criticized in that it implies that elastic phenomena may be correlated as a function of solution viscosity. The elastic phenomena for a given polymer in solution may be a single-valued function of solution viscosity, if elasticity and viscosity are related to concentration in the same way. On this basis Fabula (*7*) was able to relate the drag reduction of a series of polyethylene oxide (Polyox)–water solutions to the intrinsic viscosity, which is a function of molecular size in solution. Correlations of this type, however, have not been shown to be generally applicable to different polymer-solvent systems.

One of the properties of viscoelastic liquids which is frequently measured is the normal stress difference in laminar shear flow. Normal stress differences in laminar flow through a capillary tube are of greatest importance, since capillary-tube flow can yield the high shear rates of interest in turbulent drag reduction. The normal stresses in the axial (P_{xx}), radial (P_{rr}), and tangential ($P_{\theta\theta}$) directions are probably all different in the flow of a viscoelastic fluid. The difference usually measured is ($P_{xx} - P_{rr}$), the first normal stress difference.

Shertzer and Metzner (*8, 9*) measured this normal stress difference for Dow's J-100 and Enjay's Vistanex L-100 (polyisobutylene) solutions by measuring the jet thrust of the polymer solutions issuing from a capillary. The method has a serious shortcoming in that significant normal stress values cannot be measured for dilute polymer solutions (below 0.1%), where a high degree of drag reduction has been observed by several investigators [Hershey and Zakin (*10, 11*), Fabula (*7*), Elata et al. (*12, 13*), Lindgren (*14*), Meyer (*15*), and Ernst (*16*)], because the viscosities of dilute solutions are low, giving high Reynolds numbers (in the turbulent regime) at measurable thrusts. Shertzer (*8*) reported unusual effects in ($P_{xx} - P_{rr}$) measurements above a Reynolds number of 1,000.

Metzner and Park (*17, 18*) attempted to correlate their drag-reduction data with first normal stress difference results obtained from jet-thrust measurements. Using dimensional analysis they obtained the following groups for the correlation of the turbulent flow of a viscoelastic fluid:

$$\frac{f_{pv}-f}{f_{pv}-f_1}, N_{Re'}, \frac{(P_{xx}-P_{rr})_w}{\tau_w}$$

where f_{pv} is the friction factor for a purely viscous fluid and f_1 is on the extension of the laminar line, $(16/N_{Re'})$. These groups were used in an attempt to correlate the friction factor data for a viscoelastic 0.3% solution of J-100 in water. Metzner and Park also measured friction factors for several solutions of carboxymethyl-cellulose in water but were unable to use normal stress data for correlation because the normal stress-shear rate curves were discontinuous for different capillary tubes.

Astarita (*19*) proposed a method of correlating turbulent drag-reduction data based on the relaxation time of the polymer solution. He asserted that if the relaxation time is longer than the reciprocal of the lowest dissipative frequency in the turbulent flow, drag reduction will result. Following the development of Levick (*20*), Astarita approximated the lowest dissipative frequency in pipe flow to be $(U/2a)N_{Re}^{1/2}$. The frequencies predicted are well above the measured frequencies of maximum dissipation for the liquids studied here. Astarita reasoned that drag reduction takes place because the dissipative frequency range of the energy spectrum becomes conservative, as does a Maxwell model when stressed at frequencies higher than the reciprocal relaxation time. He further reasoned that as the total energy dissipation approaches laminar flow dissipation, the turbulent velocity profile must become steeper. There is no conclusive experimental evidence to support this contention.

Hershey (*10*) demonstrated the relationship between calculated relaxation times of the polymer solution and the experimentally determined onset of drag reduction in turbulent flow. Using the theory of Zimm (*21*), Hershey estimated the relaxation times (first five modes) for the polymer molecules in each of his solutions. The reciprocals of these relaxation times were then compared with the wall shear rates at incipient drag reduction in turbulent flow. It was found that better than "order-of-magnitude" agreement was obtained between these two reciprocal times, thus providing a basis for predicting the presence or absence of drag reduction based on the molecular weight, intrinsic viscosity, steady flow viscosity, and polymer concentration. The time scale for turbulent flow used by Hershey, the wall shear rate, can be a rough estimate of the time scale of the dissipative eddies in the turbulent stream.

Fabula, Lumley, and Taylor (*22*) have tested a hypothesis very similar to that used by Hershey. They found that the product of the first Zimm relaxation time and the wall shear rate at incipient drag reduction was nearly constant for data obtained by Virk (*23*). Ram, Finkelstein, and Elata (*24*) also tested this hypothesis and found small variation in the product for their data.

Rodriguez, Zakin, and Patterson (*25*) have demonstrated the correlation of drag reduction with viscoelasticity by plotting the friction factor ratio (measured friction factor to purely viscous friction factor) vs. a modified Deborah number $[(\tau_1 U/D^{0.2})(4[\eta]-1)]$ for several concentrations of several different polymers in organic solvents. All drag reduction data obtained by Rodriguez and by Hershey (*10*) were fit by one line, but data obtained by Elata, Lehrer, and Kahanovitz (*12*) and by Toms (*26*) required separate lines, even though each was separately well correlated.

VISCOELASTIC MODEL FOR DRAG REDUCTION

In this and later sections a quantitative expression for the reduction of turbulent energy dissipation during drag reduction is developed and tested by means of normal stress data measured with a jet-thrust apparatus. The experimentally observed effects of pipe diameter, polymer type, degradation, and velocity will be discussed in terms of the proposed mechanism.

A linear Maxwell model was assumed for simplicity to approximate the viscoelastic response of the polymer solution under shear. It was also assumed that for lengths of time of the same order as the relaxation time of the solution the shear stress on a fluid element effectively acts in only one direction. This allowed the use of the following shear-stress-direction–oriented equation:

$$s = s_g + s_\mu = \tau/G + (1/\mu)\int_o^t \tau\, dt \qquad (1)$$

If the shear stress is a cosine function of time, $\tau = A \cos \omega t$, substitution into Equation (1) gives

$$s = s_g + s_\mu = (A/G)\cos \omega t + (A/\mu\omega)\sin \omega t \qquad (2)$$

where $s = d(x - x_o)/dy$. The energy dissipation per unit volume equals $\int_o^{s_\mu} \tau\, ds_\mu$. Differentiation of Equation (2) with respect to time and substitution into the integral yields the equation for viscous dissipation for the Maxwell model:

$$W = A^2\ \omega/2\mu$$

For a purely viscous fluid, the energy dissipation would be

$$W_s = B^2\ \omega/2\mu_s$$

if $s = (B/\mu_s\omega)\sin \omega t$.

For equal root-mean-square displacement amplitudes, the viscoelastic model and purely viscous model may be related:

$$(B/\mu_s\omega)^2 \sin^2\omega t = (A/G)^2 \cos^2\omega t + (A/\mu\omega)^2 \sin^2\omega t + (A^2/\mu G\omega)\sin \omega t \cos \omega t$$

After time averaging, this relation yields

$$A^2 = B^2\ G^2\ \mu^2/\mu_s^2\ (G^2 + \mu^2\ \omega^2)$$

So the ratio of viscoelastic to viscous dissipations at equal root-mean-square displacement is

$$W/W_s = \mu/\mu_s\ (1 + \mu^2\ \omega^2/G^2) \qquad (3)$$

The term in the Maxwell model involving the viscosity represents the energy dissipative component of the model, and the term involving shear rigidity is the *nondissipative* (conservative) term. For a given deformation history, the energy dissipation may then be calculated if the viscosity and rigidity are known. For a purely viscous material $(G \to \infty)$ the entire deformation contributes to energy dissipation, but for a viscoelastic material the part of the deformation described by τ/G is recoverable; hence recoverable shear. For a steady shear flow (laminar) the elastic deformation is at equilibrium after reaching the value dictated by the shear stress. The viscous deformation increases until the shear stress is removed. Upon removal of the shear stress the conserved elastic deformation relaxes. It is this recovery of elastic deformation that lowers the amount of energy dissipation and the rate of momentum transfer in a given volume of turbulent flow. Since the shear stress on each element of fluid is continually impressed and relaxed, the elastic potential energy and the associated shear deformation are recovered at each relaxation.

For application of this relationship to turbulent flow, the spectrum of shear strain fluctuation frequencies on a fluid element must be known. This would be a Lagrangian spectrum and, therefore, impossible to measure with known techniques. But using an approximate relationship between the Lagrangian and Eulerian autocorrelation coefficients (*27, 28*) one can calculate the Lagrangian energy spectrum, which expresses the distribution of frequencies of velocity fluctuation of a particle of fluid.

It is easily shown that if the Eulerian and Lagrangian correlation functions are of the same form, that is,

$$R_L(T_L) = R(\beta<u'>T_L/\bar{u}) = R(T) \qquad (4)$$

where T and T_L are Eulerian and Lagrangian time delays and β is the ratio of Eulerian to Lagrangian length scales, then the relation between Eulerian and Lagrangian energy spectra is

$$F_L(n_L) = F\,[\bar{u}n_L/(\beta<u'>)\,] = F(n)$$

or

$$n_L = n\beta<u'>/\bar{u} \qquad (5)$$

This relation will generally produce Lagrangian frequencies much lower than Eulerian frequencies, since the intensity of turbulence, $<u'>/\bar{u}$, is usually a small fraction and β has been found to be 0.2 to 0.8 (*27, 28*).

Equation (5) provides the needed relation between Eulerian and Lagrangian velocity fluctuation frequencies to determine the latter from measurements of the former. If the shear stress fluctuations occur at frequencies equal to the Lagrangian velocity fluctuations, then the viscoelastic energy dissipation is the viscous dissipation function times the ratio of viscoelastic to purely viscous dissipation, calculated from Equation (3).

The turbulent energy dissipation for isotropic turbulence may be shown to be (*29*)

$$W_s = 60\nu_s\,(\pi<u'>/\bar{u})^2 \int_o^\infty n^2\,F(n)dn \qquad (6)$$

From Equation (6) the isotropic viscoelastic energy dissipation is

$$W = 60\nu\,(\pi<u'>/\bar{u})^2 \int_o^\infty n^2F(n)\,[1/(1+\omega^2\mu^2/G^2)\,]\,dn \qquad (7)$$

The frequency ω in radians must be the Lagrangian frequency in radians:

$$\omega = 2\pi\ n_L = 2\pi\ n\beta<u'>/\bar{u}$$

Thus the ratio of viscoelastic dissipation to purely viscous dissipation is

$$\frac{W}{W_s} = \frac{\nu \int_o^\infty [n^2F(n)/(1+4\pi^2n^2\beta^2<u'>^2\mu^2/\bar{u}^2G^2)\,]\,dn}{\nu_s\int_o^\infty n^2F(n)\ dn} \qquad (8)$$

This relationship describes a turbulent-energy-dissipation reduction mechanism which involves the following assumptions:

1. The Lagrangian and Eulerian spectra have the relationship described by Equation (5).
2. β, if not a true constant, shows only small variation with tube size, radial location, flow rate, and fluid properties.
3. The shear stress fluctuation frequencies are approximately equal to the Lagrangian velocity fluctuation frequencies.
4. The polymer-solution viscoelasticity may be adequately described for this purpose by a Maxwell model.
5. The ratio of nonisotropic purely viscous dissipation to nonisotropic viscoelastic dissipation in turbulent tube flow is nearly equal to the same ratio for isotropic turbulence.
6. The values of $<u'>$ are the same for both viscoelastic and purely viscous turbulence.

In order to test the validity of the proposed mechanism for turbulence-energy-dissipation reduction, we had to postulate an approximate relation between turbulence-energy-dissipation reduction and wall-shear-stress reduction. This was possible because the reduction of turbulent momentum flux (turbulent shear stress) in elastic fluids is apparently caused by the recovery of elastic-fluid shear displacement. This is a phenomenon directly related to the recovery of elastic shear energy described in Equation (3).

The relation between turbulent-energy dissipation and wall shear stress in the tube flow of purely viscous fluids will be assumed to hold for drag reduction in viscoelastic fluids. The wall shear stress for turbulent pipe flow is related to the Reynolds stress and velocity gradient as follows:

$$\tau_w{}^t\left(\frac{r}{a}\right) = \rho\overline{u'v'} + \mu\left(\frac{\partial\bar{u}}{\partial r}\right)$$

Away from the wall where $\partial\bar{u}/\partial r$ is small:

$$\tau_w{}^t \approx \rho\overline{u'v'}\left(\frac{a}{r}\right)$$

Since the turbulence-energy production rate, $\rho\overline{u'v'}(\partial\bar{u}/\partial r)$, is nearly equal to the turbulent-energy dissipation except very near the pipe wall or pipe center,

$$\tau_w{}^t \approx \left(\frac{W}{d\bar{u}/dr}\right)\left(\frac{a}{r}\right)$$

So the ratio of friction factors for a viscoelastic fluid to a purely viscous fluid of the same viscosity is

$$\frac{\tau_w{}^t}{(\tau_w{}^t)_s} \approx \left(\frac{W}{W_s}\right)\left(\frac{d\bar{u}}{dr}\right)_s \Big/ \left(\frac{d\bar{u}}{dr}\right) \qquad (9)$$

The change of $d\bar{u}/dr$ for drag reduction is unfortunately not known and cannot be measured by conventional techniques (*30*).

The wall shear stress in the turbulent flow of nondrag-reducing fluids is approximately proportional to Reynolds number to the 1.8 power. In laminar flow $\tau_w{}^1 \propto \mu U/D$. The ratio of turbulent wall shear stress to laminar wall shear stress is then proportional to $N_{Re}{}^{0.8}$.

It was observed that for energy spectra measured in this investigation the isotropic dissipation, $\int_o^\infty n^2F(n)dn$, was approximately proportional to bulk mean velocity. Since $<u'>$ was approximately proportional to the bulk mean velocity, W [Equation (6)] is then proportional to νU.

The ratio of turbulent and laminar wall shear stresses is then

$$\frac{\tau_w{}^t}{\tau_w{}^1} \propto \left(\frac{UD}{\nu}\right)^{0.8} \propto \left(\frac{D}{\nu}\right)^{0.8} \left(\frac{W}{\nu}\right)^{0.8}$$

The ratio of the above ratio for the polymer solution to the solvent for the same diameter yields

$$\frac{(\tau_w{}^t/\tau_w{}^1)}{(\tau_w{}^t/\tau_w{}^1)_s} = \left(\frac{W}{W_s}\right)^{0.8} \left(\frac{\nu_s}{\nu}\right)^{1.6}$$

Since, for the same flow rates,

$$\tau_w{}^1/(\tau_w{}^1)_s = \nu/\nu_s$$

$$\tau_w{}^t/(\tau_w{}^t)_s = (W/W_s)^{0.8}\,(\nu/\nu_s)^{0.6} \qquad (10)$$

Combining Equations (9) and (10) indicates that the ratio of velocity gradients at the same points in drag reducing and purely viscous fluids of the same viscosity is

$$\left(\frac{d\bar{u}}{dr}\right) \Big/ \left(\frac{d\bar{u}}{dr}\right)_s \approx (W/W_s)^{0.2}$$

Reliable experimental data for velocity profiles in drag reducing flow are needed to confirm this relationship.

APPLICATION OF THE VISCOELASTIC MODEL

Although a linear Maxwell model was used in the energy-dissipation analysis, it was not assumed that constant values of G and μ would adequately represent the fluid behavior. Normal stress difference and shear stress–shear rate data over a large shear rate range were used to estimate values of G and μ. Philipoff (*31, 32*) showed that the constancy of G is not necessary for its evaluation from normal stress difference when he derived the following equation from the strain energy of the fluid:

$$P_{xx} - P_{rr} = Gs_g^2$$

Since s_g, the recoverable shear strain, is defined by τ_{rx}/G,

$$G = \tau_{rx}^2/(P_{xx} - P_{rr}) \qquad (11)$$

The turbulent-energy-dissipation ratio of Equation (9) has been computed for several radial positions in both the 1- and 2-in. tubes for three polymer solutions (1.0% polyisobutylene L-80 in cyclohexane, 0.42% polyisobutylene L-200 in toluene, and 0.38% polyisobutylene L-200 in cyclohexane). Normal stress data were obtained on the first solution by Green (*33*), and normal stress differences were measured by Patterson et al. (*34*) on fresh and degraded samples of the second and third solutions. Figure 1 shows a comparison of the normal stress $(P_{xx} - P_{rr})$ data. Least-square-straight-line fits of the polyisobutylene L-200 in toluene data were used because of the unusual curvature at low shear rates. Values of shear rigidity modulus, G, were calculated by means of Equation (11) and are shown along with values of viscosity, μ, in Table 2.

Turbulence intensities for the locations close to the wall were estimated from Laufer's data for air in a 10-in. pipe (*35*). Since turbulence intensity measurements were not possible very near the wall in this investigation, the effect of drag reduction on intensity near the wall was not known. No attempt was made, therefore, to account for this effect. Calculations were made for turbulent-core locations and for locations as close to the wall as $y^+ = 12$, the point of maximum energy dissipation according to Laufer's data. Use of measured intensity results from a close-wall probe in drag-reducing solutions will improve the accuracy of the calculations. The turbulence intensities, average velocities, and W_s values used in each pipe are shown in Table 1.

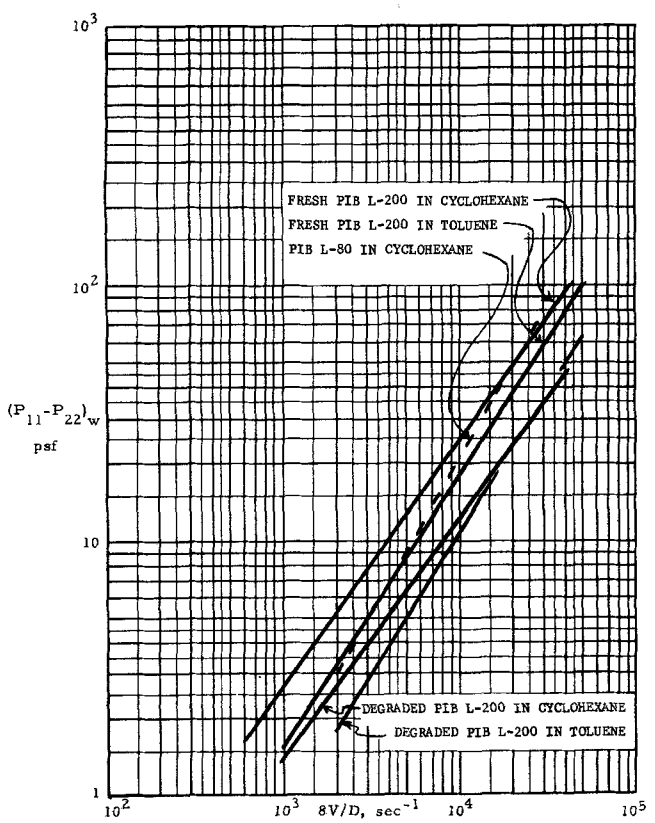

Fig. 1. First normal stress difference.

Table 1. Velocities and Turbulence Intensities Used in Drag Reduction Mechanism Calculations

Tube size, in.	Space average velocity, ft./sec.	y^+	$\langle u' \rangle / \bar{u}$	$\bar{u}$, ft./sec.	$W_s \times 10^4$,† sq.ft./sec^3
1.0	18	12*	0.28	9.0	477
		24*	0.20	11.5	244
		48*	0.15	13.7	137
		100	0.13	16.0	103
		300	0.10	18.2	61.0
		1,000	0.04	20.5	9.8
		2,000	0.03	22.8	5.5
1.0	6.6	12*	0.30	3.14	230
		24*	0.22	4.29	124
		48*	0.17	5.11	74.0
		100	0.15	5.76	57.5
		300	0.12	6.60	37.0
		1,000	0.05	7.42	6.4
		2,000	0.03	8.25	2.3
2.0	5.5	12*	0.35	2.62	131.0
		24*	0.25	3.59	67.0
		48*	0.20	4.28	42.9
		200	0.12	5.51	15.4
		667	0.05	6.22	2.7
		1,334	0.03	6.90	1.0
2.0	2.8	12*	0.40	1.32	88.0
		24*	0.30	1.81	49.5
		48*	0.25	2.16	34.4
		200	0.15	2.78	12.4
		667	0.06	3.14	1.9
		1,334	0.04	3.48	0.9

* Laufer (35).
† Equation (6), with toluene viscosity.

The shear rates used to calculate viscosities and normal stresses were calculated for use in Equation (8) as follows:

$$\langle ds/dt \rangle = \langle du'/dy \rangle = \sqrt{W/\nu}$$

$$= \left[60\pi^2 \langle u' \rangle^2 \int_o^\infty n^2 F(n)\, dn \right]^{1/2} \Big/ \bar{u} \qquad (12)$$

The root-mean-square turbulent shear rates obtained from this equation are only an approximation of the true shear rates experienced by the polymer solution. The instantaneous shear rate calculated for nonisotropic turbulence would yield a more rigorous result. This was impossible with the present knowledge of turbulence. Nevertheless, as shown below, the final results are still close to measured drag ratios.

Wall viscosities were calculated for use in Equation (10) by a trial-and-error method.

Energy spectra used in Equations (6) and (12) were reported by Patterson and Zakin (*36, 37*), who found that tube size and solution properties (elastic and viscous) made little difference in the energy spectra. Therefore Newtonian solvent spectra were used for these calculations.

The calculated results from using the proposed mechanism of turbulent-energy-dissipation reduction and its ap-

Table 2. Turbulent-Energy-Dissipation Ratios and Drag Ratios Calculated from a Viscoelastic Mechanism
(All values for $y^+ = 12$, for $\beta = 0.035$.)

Tube size in.	Space average velocity, ft./sec.	Polymer in solution	Solvent	Solution viscosity, μ, centipoises	Rigidity modulus, G, lbf/sq.ft.	RMS shear rate [Eq. (12)], sec.$^{-1}$	W/W_s	Calculated drag ratio $(\tau_w{}^t)/(\tau_w{}^t)_s$	Measured drag ratio $(\tau_w{}^t)/(\tau_w{}^t)_s$
1.0	18	L-80	Cyclohexane	4.71	0.0273	4903	3.73	1.06	1.21*
1.0	6.6	L-80	Cyclohexane	5.26	0.01585	1820	5.08	1.26	1.55*
1.0	18	Used L-200	Toluene	1.64	0.00588	4903	1.72	0.78	0.84
1.0	6.6	Used L-200	Toluene	1.71	0.00265	1820	2.26	0.94	1.0†
1.0	18	Fresh L-200	Toluene	1.84	0.00421	4903	1.42	0.62	0.70
1.0	6.6	Fresh L-200	Toluene	1.99	0.00192	1820	2.02	0.78	0.9†
1.0	18	Used L-200	Cyclohexane	3.10	0.01639	4903	2.35	0.94	0.89
1.0	6.6	Used L-200	Cyclohexane	3.47	0.00714	1820	3.00	1.06	‡
1.0	18	Fresh L-200	Cyclohexane	4.33	0.01579	4903	2.68	0.85	0.70
1.0	6.6	Fresh L-200	Cyclohexane	5.01	0.00758	1820	3.82	1.04	‡
2.0	5.5	L-80	Cyclohexane	5.70	0.01063	878	5.96	1.37	1.50*
2.0	2.8	L-80	Cyclohexane	6.46	0.00574	285	7.10	1.46	1.59*
2.0	5.5	Used L-200	Toluene	1.77	0.00148	878	2.64	1.04	‡
2.0	2.8	Used L-200	Toluene	1.86	0.00060	285	3.12	1.16	‡
2.0	5.5	Fresh L-200	Toluene	2.10	0.00108	878	2.52	0.90	‡
2.0	2.8	Fresh L-200	Toluene	2.30	0.00044	285	3.33	1.07	‡
2.0	5.5	Used L-200	Cyclohexane	3.78	0.00388	878	3.52	1.15	‡
2.0	2.8	Used L-200	Cyclohexane	4.31	0.00151	285	4.30	1.24	‡
2.0	5.5	Fresh L-200	Cyclohexane	5.58	0.00442	878	4.76	1.16	‡
2.0	2.8	Fresh L-200	Cyclohexane	6.58	0.00192	285	6.31	1.31	‡

* Hershey (10).
† Extrapolated from data for a lower concentration.
‡ Not measured, but estimated to be 1.0 or slightly higher.

proximate relation to wall shear stress are compared with experimental results in Table 2. W/W_s values for Equation (10) should be integral values for the entire pipe cross section, but since energy dissipation at $y^+ = 12$ is dominant (see Table 1), integration over the entire tube cross section caused only small changes in W/W_s from values calculated for $y^+ = 12$. Drag ratios were estimated by using the W/W_s values at $y^+ = 12$. W/W_s values were obtained by setting the value of β at 0.035. This adjustment from measured values of 0.2 to 0.8 for air turbulence was necessary to obtain correct absolute-drag-ratio levels. The necessity of using such a low value for β indicates that the dissipation ratio overestimates the elastic effect; that the magnitude of the shear rigidity, G, is too low; or that the ratio of Eulerian to Lagrangian scales is lower in viscoelastic turbulence.

Discrepancies in the comparison of calculated and observed drag ratios—the low calculated values for polyisobutylene L-80 in cyclohexane for both pipe sizes and the high calculated value for fresh polyisobutylene L-200 in cyclohexane in the 1-in. pipe at 18 ft./sec.—may be attributed to a number of possible weaknesses in the theoretical treatment:

1. The Lagrangian and Eulerian energy spectrum relationship of Equation (5) is only a crude empirical approximation. The changes of solution properties when different polymers or solvents are used could cause variations in β.

2. Root-mean-square shear rates were used to evaluate the values of G from normal stress data. The use of G as a function of shear rate rather than a single value would have been more realistic but was not justified by the approximate relationship between G and normal stress difference.

3. The effect of drag reduction on turbulence intensity was neglected because of lack of a definitive relationship between them. [Drag reduction has been shown in some cases to be accompanied by an increase in turbulence intensity (36, 38).]

4. The assumption that the anisotropic and isotropic dissipation ratios are nearly equal is merely the best available approximation. Some error is bound to arise from this assumption.

Inconsistencies in polymer degradation levels undoubtedly contributed to the discrepancies in Table 2.

Since two flow rates were used in each of two tube sizes for five polymer systems, the degree of correlation with measured values indicates that the assumptions and approximations involved were reasonable. Calculated drag ratios were generally slightly lower than measured values except for polyisobutylene L-200 in cyclohexane, but the trends with velocity, pipe size, and solution properties were all very close to measured trends. This mechanism shows quantitatively the relative importance of solution properties (viscosity and shear rigidity modulus) and flow conditions (velocity, turbulence intensity, and pipe size). A relatively viscous solution may be very friction-factor reducing [friction-factor ratio may be calculated by using the solution viscosity in place of solvent viscosity in Equations (8) and (10)], if it has a long relaxation time, yet not be drag reducing at a given flow rate because of the high ratio of its viscosity to that of the solvent.

The diameter effect observed for drag reduction (greater reduction in smaller tubes at the same Reynolds number) occurs because of the small effect of pipe size on spectrum frequencies at the same space-average fluid velocity.

CONCLUSIONS

The above viscoelastic model demonstrates that drag reduction may be predicted without the assumption of a reduction of turbulence intensity. The reduction in the turbulent-momentum-transfer rate calculated at the same turbulence intensity suggests that turbulent heat and mass transfer may be reduced in the same manner in viscoelastic

fluids without a decrease in turbulence intensity. This has serious implications in the design of heat exchangers and reactors involving viscoelastic fluids in turbulent flow.

The success of the empirical relationship between turbulent-energy-dissipation reduction (derived from one-dimensional energy spectra) and the wall-shear-stress reduction depends on the close relationship of turbulent-energy dissipation and turbulent-momentum transfer. This relationship was used empirically to calculate drag reduction from turbulent-energy-dissipation reduction.

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NOTATION

a = pipe radius
A = amplitude constant
B = amplitude constant
D = tube or pipe diameter
$F(n)$ = Eulerian energy spectrum
$F_L(n_L)$ = Lagrangian energy spectrum
f = measured friction factor, $(D\Delta P/4L)/(\rho U^2/2g_c)$
f_{pv} = friction factor predicted for a purely viscous fluid
f_l = friction factor on the extension of the laminar line, $16/N_{Re'}$
g_c = force-to-mass conversion factor
G = shear rigidity modulus
K' = generalized consistency factor, $\tau_w = K'(8U/D)^{n'}$
n = Eulerian frequency, cycles/sec.
n_L = Lagrangian frequency, cycles/sec.
n' = flow index in $\tau_w = K'(8U/D)^{n'}$
N_{Re} = Reynolds number, $\rho UD/\mu$
$N_{Re'}$ = generalized Reynolds number, $\rho U^{2-n'}D^{n'}/g_cK'8^{n'-1}$
P_{xx}, P_{rr}, $P_{\theta\theta}$ = normal stresses in the x, r, and θ directions
r = radial distance from pipe center
$R(T)$ = Eulerian autocorrelation coefficient
$R_L(T_L)$ = Lagrangian autocorrelation coefficient
s = shear strain in any direction, $d(x - x_o)/dy$
s_g = elastic component of shear strain
s_μ = viscous component of shear strain
t = time
T = Eulerian delay time
T_L = Lagrangian delay time
u = instantaneous velocity in axial direction
$\bar{u}$ = time average velocity in axial direction
u' = fluctuating velocity in axial direction, $u - \bar{u}$
$\langle u' \rangle$ = root-mean-square fluctuating velocity
u^* = friction velocity, $\sqrt{\tau_w/\rho}$
U = bulk mean velocity
v' = fluctuating velocity in radial direction
W = turbulent-energy-dissipation rate
W_s = turbulent-energy-dissipation rate in the pure solvent
x = direction of shear strain
y = direction transverse to shear strain
y^+ = dimensionless distance from pipe wall, $\rho u^* y/\mu$

Greek Letters

β = constant in Equation (4)
$[\eta]$ = intrinsic viscosity
μ = fluid viscosity
μ_s = solvent viscosity
μ_o = zero shear rate viscosity
ν = fluid kinematic viscosity
ν_s = solvent kinematic viscosity
π = ratio of circumference to diameter for a circle
ρ = fluid density
τ = shear stress on a fluid element
τ_1 = first mode relaxation time in Zimm theory
$\tau_{1/2}$ = shear stress where the viscosity is $\mu_o/2$
τ_w = wall shear stress
$\tau_w{}^l$ = laminar wall shear stress
$\tau_w{}^t$ = turbulent wall shear stress
ω = frequency, radians/sec.

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